

Article

A Novel Approach to Integrating Uncertainty into a Push Re-Label Network Flow Algorithm for Pit Optimization

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Abstract: The standard optimization of open-pit mine design and production scheduling, which is impacted by a variety of factors, is an essential part of mining activities. The metal uncertainty, which is connected to supply uncertainty, is a crucial component in optimization. To address uncertainties regarding the economic value of mining blocks and the general problem of mine design optimization, a minimum-cut network flow algorithm is employed to give the optimal ultimate pit limits and pushback designs under uncertainty. A structure that is computationally effective and can manage the joint presentation and treatment of the economic values of mining blocks under various circumstances is created by the push re-label minimum-cut technique. In this study, the algorithm is put to the test using a copper deposit and shows similarities to other stochastic optimizers for mine planning that have already been created. Higher possibilities of reaching predicted production targets are created by the algorithm's earlier selection of more certain blocks with blocks of high value. Results show that, in comparison to a conventional approach using the same algorithm, the cumulative metal output is larger when the uncertainty in the metal content is taken into consideration. There is also an additional 10% gain in net present value.

Keywords: open-pit mine; ultimate pit limit; uncertainty modeling; minimum cut; network flow

MSC: 68W40; 68W50



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1. Introduction

A challenging issue in mine planning and design is open-pit mine production scheduling. To maximize the total discounted profit, an open-pit optimization problem is used to remove mining blocks from the pit while satisfying all of its constraints. Reserve constraints: the restriction that a block can only be mined once during its lifetime. Slope constraints: a block cannot be mined before its predecessors. A group of overlying blocks must be removed to reach a given block. Mining constraints: to ensure the effective use of mining equipment, the total weights of blocks mined during each period should be at least equal to a minimum mining limit. On the other hand, it should not be greater than the capacity of the mining equipment that was in use at the time. Both the upper bound and the lower bound must be satisfied under mining constraints. Processing constraints: this primarily depends on the processing capacity of the plants. The total number of ore blocks mined during each period should at least be equal to the minimum number needed for processing, but it should not exceed the processing plant's capacity because the excess ore must then

be stored somewhere. Metal production constraints: Both the upper and lower bounds of the metal production limitations must be satisfied, and the amount of metal recovered from the ore blocks processed should not be less than a minimum amount and should not exceed the amount that can be sold during this period. Open-pit optimization is typically defined as a type of integer or mixed integer programming issue that can be solved utilizing a commercial solver. A three-dimensional array of blocks is used to depict the orebody. With the limited number of exploration drilling data and the use of geostatistical algorithms, the mineral grade of each block within the orebody is determined. The volume, grade, and tonnage details are provided for each block. It is well recognized that orebody models and their geological features are a significant source of scheduling risk. Every block has a block economic value that represents the corresponding net profit. The challenge of allocating a mining block's extraction sequence to various production periods is the main focus of open-pit mine design. A mining block's weight, block economic value, ore contents, metal contents, and other characteristics are used to describe it. Many mining blocks are assigned to distinct production periods to maximize profit within predetermined bounds. These restrictions not only include, but also contain: (a) a block must be allocated to one production period; (b) a set of neighboring blocks for each block must have the same production period or prior periods; and (c) a sufficient quantity of blocks must be assigned during a specified time frame to ensure that the total quantity of material, ore, and metal recovered from the mining falls within the allowable limit for a particular production period. An open-pit mine's production schedule is a challenging issue [1–3]. Usually, mixed integer programming is used to resolve open-pit production scheduling [4]. Finding the best production schedule is a challenging task because of the huge number of integer variables, numerous limitations, and lack of certainty around the various mining block parameters. The main source of uncertainty is geological, which implies that the metal composition of a block has been accurately determined previously. Additionally, the economic fluctuation of metal prices may cause uncertainty [5]. Numerous optimization techniques have been suggested to solve the deterministic production scheduling while treating the mining block values as known and ignoring their uncertainty. Mixed-integer linear programming methods have been proposed to optimize the net present value for many mining projects [6,7]. An alternative approach built on Lagrangian relaxation was suggested by Dagdelen and Johnson [1]. For determining the production plan of an open-pit mine, Caccetta and Hill suggested a branch-and-cut approach [8]. The Fundamental Tree Algorithm (FTA), created by Ramazan [9], decreases the size of the real problem by aggregating mining blocks. To address the mine production scheduling issue, Bley et al. [10] suggested a technique based on the cutting-plane method. For enhancing the production scheduling problem's computational effectiveness, Bienstock and Zuckerberg [11] presented a heuristic method. A mine planning framework based on network flow is presented by Topal and Ramazan [12], who also illustrate a significant application in a real mine. Using a heuristic-based linear relaxation, Chicoisne et al. [13] tackled a bigger size (3 million blocks) mine process optimization issue. A solution based on sliding windows has been suggested to address deterministic mine design [14,15]. Mining block sequencing issues are addressed by Lambert and Newman [16] using a Lagrangian relaxation-based strategy. Lamhgari et al. [17] use metaheuristic techniques to handle production scheduling in open-pit mines, such as variable neighborhood descent. It takes a lot of effort to incorporate mining waste management and removal into the MILP optimization of the project plan [18]. The mining sector has been using deterministic models for open-pit optimization since the 1980s [19–23]. Unfortunately, the basic presumption that grade and quantity will always be constant in a given block oversimplifies the issue and results in an inaccurate evaluation [24–27]. The block grades, and subsequently the metal contents, are computed using a finite number of samples, resulting in usual uncertainties with these usually expected values [28]. Numerous simulated orebody models are created using geostatistical modeling techniques to account for the geological uncertainty regarding block grades and metal concentration [28–31]. Therefore, when including these uncertainties, the stochastic

approach is more accurate. By maximizing the net present value and reducing the variance from the target, some authors proposed uncertainty-based approaches based on the concept of geological risk discounting [15,32]. A multi-stage stochastic programming approach was investigated by Boland et al. [33] to handle different aspects of mine production scheduling.

However, Godoy and Dimitrakopoulos [25] suggested a different strategy that would utilize a simulated annealing process to minimize the computing time. To optimize a mine schedule, researchers used a series of pit shells created by the layered application of the Lerchs and Grossmann approach [34–36]. They have demonstrated that their techniques are computationally faster than the conventional method, boosting the project value by 15% to 28% [34,35]. For resolving the computing challenges related to full-scale stochastic integer programming, Lamghari and Dimitrakopoulos [17] suggested a metaheuristic approach and created an algorithm combining Tabu search. For stochastic production planning, a two-stage stochastic integer programming method with a stockpiling choice and geological risk discounting is proposed [37]. The preceding model is expanded to include mining complexes [27,38]. To handle production schedules as well as waste dumping, Rimélé et al. [39] adopt a two-stage stochastic integer programming method.

In this study, the push re-label minimum-cut method is improved and tested in terms of the design of pushbacks and optimal pit bounds under uncertainty. The latter is produced by parameterizing the minimum-cut graph's arc capacities. Similar to the commercial Lerchs and Grossmann algorithm implementations, the time value of money and the associated discounting of block economic values are implemented indirectly and are based on bench-wise scheduling and predefined mining capacity. A test case on a copper mine serves to illustrate the procedure's complexity. It should be noted that the case study maintains generality by using numerous economic values for mining blocks that were produced from simulated realizations of the resource. Combining simulated orebody models, price and exchange rate forecasts cost forecasts, and other forecasts could produce economic values. The method given here is compared against the comparable deterministic variant, using only deterministic inputs, in this comparative research. The push re-label minimum-cut algorithm is initially described in the following sections about mine design and optimization. The push re-label minimum-cut approach is described after that in a brief explanation. Some similarities to the deterministic scenario are presented.

The paper continues with a literature survey. The solution approach is discussed in the part titled "Methodology," and its application to a copper deposit is covered in the section titled "3.1. A minimum-cut graph to optimize ultimate pit limits, 3.2. Uncertainty implementation in a minimum-cut graph, 3.3. Pushback design using arc capacity parameterization in a minimum-cut graph, 3.4. A mathematical formulation of a stochastic mine pit optimization under uncertainty." The paper is finished with the sections "Results" and "Conclusion and future scope."

2. Literature Survey

According to the number of highly possible scenarios, metal and geological uncertainties are treated by utilizing a spatial stochastic modeling approach to mine maps of grade, metal content, and geology. The spatial correlation among observations from the drilling procedure and the volumetric variances between the available data and the supposed mining blocks are explicitly taken into account by these algorithms [40–42]. By increasing the extraction duration of the high-risk blocks, the authors created the hypothesis of orebody risk discounting, which postpones the risk for future timeframes [43]. The main issue with the abovementioned considerations is that they fail to take into account simulation results, a risk that has been simultaneously analyzed as sets of blocks in mine from period to period, and previously given risk probability for each block [43,44]. Ramazan and Dimitrakopoulos provided the first mathematical models that successfully depicted the stochastic characteristics of mine design [45]. They investigate a two-stage stochastic model with multiple equivalent possibilities to represent unpredictable geology. Each scenario has two parts: the first phase addresses the mining sequence, and the second phase addresses any variation

from the target. These two phases make up the decision-making mechanism. The goal is to increase the expected net present value as much as possible while minimizing production target deviations, which reduces the risk that production targets will not be met.

The idea of simulated annealing was initially developed [25] and further investigated [34,46] for long-term production schedules in a stochastic approach. This approach was enhanced to take into account various mines, inventories, and operational sites [27]. A stochastic integer linear programming approach was created by Ramazan and Dimitrakopoulos [37] to optimize the net present value while reducing the deviation from production goals. A test case using the stochastic approach under risk was reported by Chatterjee and Dimitrakopoulos [46]. This makes it more challenging, but also more advantageous, to include geological risk in the optimization method. These advantages were first emphasized [25,34,37,46–48].

Geostatistical techniques can be used to measure, model, and quantify uncertainty. This is accomplished by utilizing any geostatistical simulation technique to generate numerous equal probability scenarios of orebody realization. Risk can be minimized by incorporating uncertainty into decision-making processes. As a result, the team responsible for mine design and production schedules will be able to achieve higher profit margins and develop a more effective risk management plan. The mining sector, on the other hand, is well conscious of the uncertainties and risks. Stochastic models receive significant attention since they manage uncertainty and risk in a way that is more practical and realistic. Evaluating the revenue under different scenarios and attempting to reduce production target deviations is a successful strategy for resolving stochastic problems [34,46,48].

Since geological uncertainty directly affects the supply of ore and metals, mining industries believe that it is the main cause of their unrealized cash flow projections. The challenge concerning how to determine the supply of ore for processing arises. It is a complicated issue to answer since it depends on the removal sequence throughout time in addition to the ore's spatial variability. The same resource will yield various ore supplies depending on the extraction methods used. Because it depends on economic factors and changes over time, the definition of ore changes over time. The concept of accessible ore availability has typically been assessed under the presumption of constant technical and financial limitations in both space and time. Throughout the traditional mine production and design framework, the aggregate form of ore grade assumption is utilized in combination with geological, financial, and ecological limitations to create the extraction schedule that yields the highest economic benefit. The utilization of risk-free models revealed a considerable gap between prediction and real financial results.

Several studies have investigated the effect of geological uncertainty on project economics and employed conditional simulations to undertake a risk analysis of mine plan technical specifications [24,49]. The existence of risk modeling techniques facilitates the development of novel scheduling systems that incorporate simulated geological uncertainties in mine strategic planning. The initial pushback design's stochastic graph closure problem is a relaxed scenario in which resource limitations are not considered. Resource limits must be applied to create pushbacks. Several techniques are addressed in the various literature. The suggested algorithm's major benefit is that it can be computationally very quick; therefore, incorporating uncertainty into production planning is regularly viable.

3. Methodology

The proposed approach is conceptually presented as (i) a minimum-cut graph to optimize ultimate pit limits.

(ii) Uncertainty implementation in a minimum-cut graph.

(iii) Pushback design using arc capacity parameterization in a minimum-cut graph.

3.1. A Minimum-Cut Graph to Optimize Ultimate Pit Limits

A simple, two-dimensional example can be used to illustrate open-pit optimization as a minimum graph cut problem. Figure 1 depicts a section where the associated economic value is represented by the numerical values inside each block. Based on the expected

grade, metal selling price, mining cost, and processing cost, a block’s economic worth is determined. Blocks with positive values are ore blocks, while blocks with negative values are waste blocks. Figure 2 illustrates a directed graph as a possible representation of this two-dimensional orebody model. The directed graph is thought of as having each mining block as a node. A source and sink are two unique nodes. The model’s ore block nodes are linked to the source node, and the capacity of those arcs represents the economic value of blocks. However, every node within the model that is defined as a waste block is linked to a sink node, and the capacities of those arcs are equal to the absolute values of waste block nodes. One needs to gain access to that specific block to mine a certain block, $x(i, j)$. It is impossible to reach a specific block and mine it unless the blocks above it are removed; this is made feasible by retaining slope limits. To meet slope limitations, it is required to identify the underlying blocks that need to be eliminated before removing the desired block. The red arrow in Figure 2 indicates a block’s slope constraints. The slope constraint arcs’ capacities are given an infinite value. Slope constraint arcs can never be in the minimum cut since their value is indefinite. As a result, the minimum cut will result in an acceptable pit because the slope constraint will not be satisfied. The optimal ultimate pit can be found by transforming the example of a two-dimensional orebody into a directed graph problem and then applying the minimum-cut method. A set of directed arcs with at least one arc in each direction from the source to the sink node forms a cut of a directed graph. There will not be a straight path between the source and sink nodes if the arcs in the cut are eliminated. The cut value is the total of all of the arcs in the cut’s flow capacity in the source-to-sink direction.

2	5	-2	2	-2
-3	6	-1	4	-2
-5	8	4	-7	-4

Figure 1. Block model with block economic values.

Finding the cut in the graph where the total of the capacities is least across all cuts is the goal of the minimum-cut problem. Figure 2 shows the minimum cut of the two-dimensional open pit that was indicated in Figure 1 with a bold, brown line. The orebody model in this example has a minimum cut value of 8. In this illustration, the ore blocks are on the sink node of the cut, and the waste blocks are on the source node. The minimum cut in the open-pit optimization issue is the cut that minimizes the sum of the number of waste blocks on the source node side and the sum of the number of ore blocks on the sink node side. In other words, the minimum cut maximizes the number of ore blocks present in the pit while simultaneously reducing the waste blocks present in the pit. Given that the slope restrictions are followed, the minimum cut is likewise a valid pit. When the capacities of the ore blocks on the sink side and the waste blocks on the source side are added together, it was determined that the cut shown has value $(3 + 1 + 2 + 2) = 8$, which is the same as this problem’s minimum cut value.

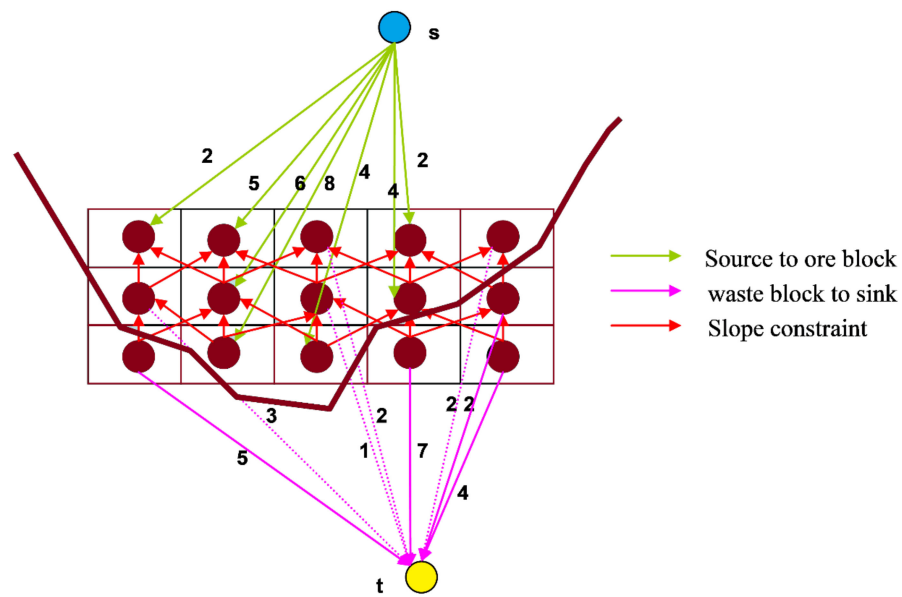


Figure 2. Pit limit calculation using the minimum directed graph cut.

3.2. Uncertainty Implementation in a Minimum-Cut Graph

Multiple orebody models can be simply added to the previously described technique. Figure 3 illustrates the same directed graph that is produced utilizing numerous orebodies, as in the previous example. All ore blocks will be linked to the same source node, and all waste blocks will be linked to the sink node. Using arcs with infinite capacity, the slope constraint will be maintained. A specific block in simulated orebody models may be wasted in one simulated block model and ore in another. Despite the computer simulation, the source node is linked to all ore blocks, while the sink node is connected to all waste blocks. This suggests that a certain block can be connected to the source node in one realization while also being connected to the waste node in another realization. The arc capacity will be different. Using the top-left block in Figure 3 as an example, let us say that the ores in Simulations 1 and 2 have economic values of 2 and 1, respectively. The source node is connected to that block in Simulations 1 and 2 with arc capabilities of 2 and 1, respectively. The same block, however, has a block economic value of -6 in Simulation 3, and an arc with an arc capacity of 6 is constructed from the block to the sink. Integrate the data from all simulations, and this is comparable to creating a directed graph with the number of nodes equal to n times the number of blocks in the deposit, where n is the number of simulated orebodies. The values of the blocks can be created using simulated grades, simulated grades paired with simulated commodity price forecasts, simulated grades combined with simulated commodity price forecasts and mining/processing expenses, etc. This allows for the integration of any uncertainty related to the estimation of a block’s value.

Multiple orebody models can be used to make the directed graph, and then the minimum-cut technique can be used to create an ultimate pit. The identical block from the various simulations may not lie on the same side of the minimum cut if the minimum-cut approach is applied to the directed graph defined above with n no. of simulations. The main difficulty in formulating the minimum-cut technique for addressing the open-pit optimization problem with simulated orebodies is that, to produce an ultimate pit, a given block from various simulations must be on the minimum-cut side. No matter how many simulations are performed, the choice should be made in terms of a block as either being in the ultimate pit or not. To guarantee that a specific block lies on the same side of the cut for all simulations, another constraint must be added to the graph. It is possible to implement this constraint by combining blocks from many simulations into a bidirectional arc with infinite capacity. This ensures that there will never be a situation in which the same block from different simulations will fall on different sides of the minimum cut because these

bidirectional arcs have infinite capacity and will never be in the minimum cut. So long as all requirements are respected, the pit produced by the minimum-cut algorithm with various orebody models is valid. These nodes can be combined into a single node because the same block will appear in the same simulation on the same side of the minimum cut. A single arc can be created by merging the arcs from the source node to the merged nodes from various simulations, and the arc's capacity will equal the total of all the capacities when that particular block is ore in all scenarios. Similar to the previous example, a single arc can be created from a merged node to the sink node, and its capacity will equal the sum of the actual capacities of the block's economic values in all simulations when the block is wasted. Figure 4 illustrates this idea. The graph there was created by integrating the three simulated orebody models from Figure 3. Each block in this figure has two values assigned to it. If the realizations are ore, the value in the top-left corner represents the total block economic values across all realizations. If any realizations are wasted, the value in the bottom-right corner represents the total relative block economic value. The value in the top-right corner represents the arc capacity from the source to that node, and the value in the bottom-left corner represents the arc capacity from the node to the sink. In Figure 4, for illustration, the top-left block's arc capacity from the source to a node is 3, which is the sum of 2 and 1, the block economic values of Simulations 1 and 2, respectively. With a capacity of 6, the absolute block value in Simulation 3, the same node is connected to the sink.

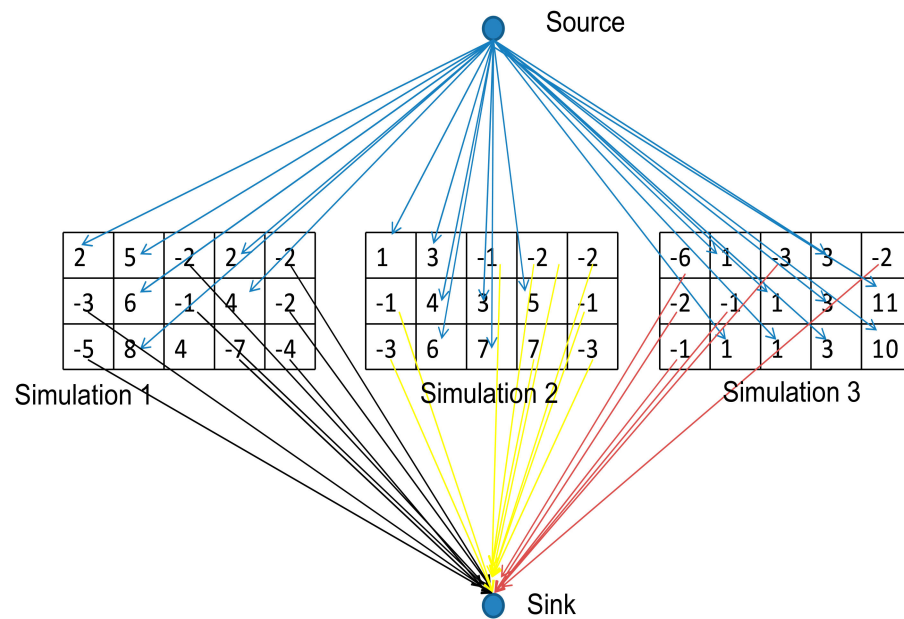


Figure 3. Multiple orebody models were used to create the graph.

The reduction in the number of nodes in the graph is the key benefit of the suggested strategy with different orebody models and economic values. Except for the source and sink nodes, the network has the same number of nodes as there are blocks. Due to this, regardless of the number of simulated orebodies, a deposit's node count will always be the same. The capacity of the arcs is the only thing that varies when the number of simulated models is altered. To take into consideration the geological risk in open-pit design, a stochastic variant of the network flow algorithm is utilized in the block economic value. To manage uncertainty, the stochastic network flow technique uses simulated models of an orebody that are equally probable. Calculations are performed using a set of scenarios rather than a precise and perhaps inaccurate model. This maximizes the net present value for an open-pit mine, given the uncertainty from limited data and orebody models. The method works as follows: the economic block value of each block in each simulation is determined when the simulations are run. The blocks with a negative block economic value are then connected to a sink that is the same for all simulations (one single sink node).

The same is done with blocks with positive block economic value; these are attached to a single source node and precedence restrictions must be followed. Furthermore, blocks with the same grid location (x, y, z) will have an endless number of capacity arcs connecting them. Because just one pit is being created, the additional constraint exists because the same blocks in the grid must be either within or outside the pit for each simulation. The algorithm then merges the blocks at the same place into a single block (node) with no more than one arc from the source node and one arc to the sink node, where these arcs have the capacity of the sum of the capacity of the arcs at the same place in the same simulation. This produces a single graph (matrix), which makes the procedure less computationally demanding because the number of nodes is drastically decreased and just one minimum-cut algorithm is required. When more simulations are added to the process, the capacity of the arcs increases considerably.

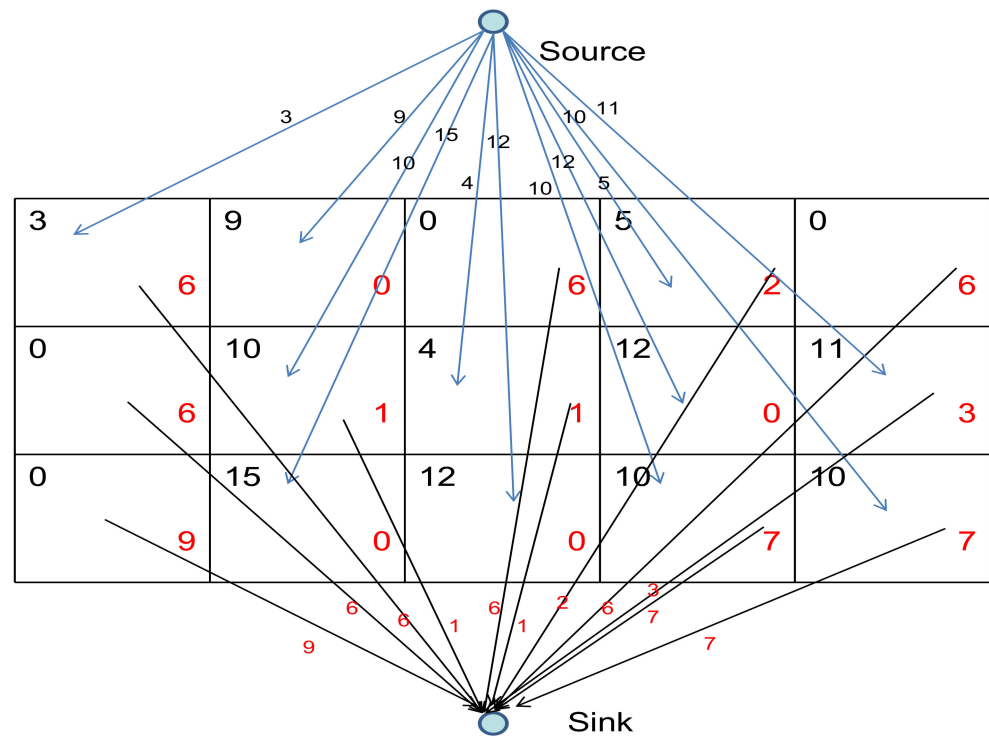


Figure 4. Three simulated models are combined to create new graphs.

3.3. Pushback Design Using Arc Capacity Parameterization in a Minimum-Cut Graph

The simulated orebody models construct a single pit, which is the ideal ultimate pit for the deposit, using the previously proposed minimum-cut network flow technique. Parameterization of the minimum-cut algorithm can be used to generate pushbacks. The Lerchs and Grossmann parameterization algorithm is well documented in the literature—for example, Seymour [50]—and is used in commercial implementations [23]. A succession of “nested” pits can be produced by scaling the economic values of all blocks with a multiplier parameter, λ . The same idea that may be used to build nested pits with a minimum-cut method can be used to parameterize the Lerchs and Grossmann algorithm.

Depending on which term is multiplied throughout the block economic value calculations, the value of λ must increase or decrease monotonically. If λ is used to immediately multiply the economic block value, rising λ values will produce pits ranging in size from small to large until the final pit limit is achieved. If λ is multiplied by the mining or processing cost, increasing values will result in larger to smaller pits. It is also feasible to utilize more than one parameter for parameterization. By changing the λ value to $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$, where $\lambda_1 < \lambda_2 < \lambda_3 < \dots < \lambda_n$, it is possible to obtain pushback $P_1, P_2, P_3, \dots, P_n$ with pit sizes $P_1 < P_2 < P_3 < \dots < P_n$. In this paper, λ was chosen as a monotonic non-decreasing value, which is multiplied by the capacity of

the arcs from the source to the ore block nodes. Because the goal is to scale the economic worth of the ore blocks, the arcs from waste block nodes to sink are left alone. Figure 5 depicts the revised graph with the parametric version.

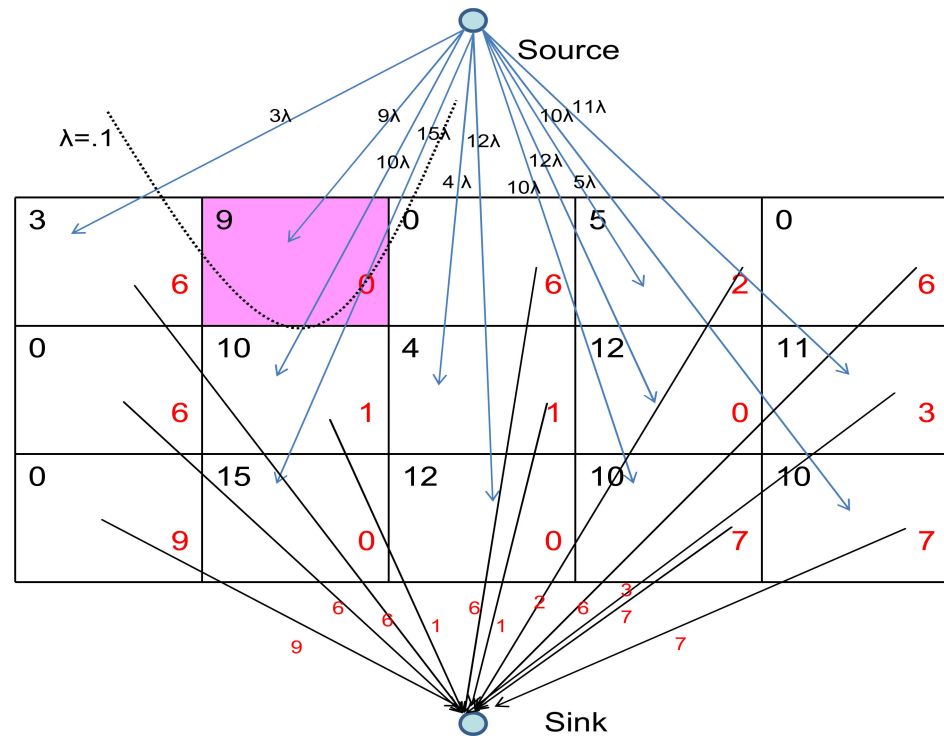


Figure 5. Parametric minimum-cut graph after multiplying λ with arcs from source to ore blocks. The first pushback is generated when the value of λ is 1.

It is obvious that the network algorithm’s parameter λ , a multiplier of the arcs’ capacity, is essential for creating nested pits or pushbacks. For the generation of nested pits, choosing a series of λ values is tough work. The random selection of λ may lead to pushbacks with huge gaps, which is unfavorable for mining. Additionally, a crucial issue is the number of pushbacks that must be created. For the sake of parameterization in this study, just one parameter is considered. When the λ value is set to 1, the ultimate pit is achieved. A sequence of nested pits is created by reducing the value of λ .

3.4. Mathematical Formulation of a Stochastic Mine Pit Optimization under Uncertainty

A two-stage stochastic mixed-integer programming model can be used to formulate the open-pit optimization issue. For each period of the horizon, a set of blocks to be mined is determined in the first stage, taking into consideration the minimum and maximum mining limits. Each block throughout each set is scheduled exactly once after all of its predecessors. The category of the block, such as ore or waste, and the amount of metal within the block are unknown at this point. For each scenario, uncertainty is addressed in the second stage. For instance, the total amount of ore needed for processing could, at times, surpass the processing plant’s capacity, while at other times, it might not satisfy the minimum requirements. This may apply to the metal recovery from the blocks of processed ore. The suggested model’s goal is to minimize the deviation from the production target while maximizing the profit for all simulations, S , by allocating N blocks over T production periods. The value of the objective function will be lower if the schedule deviates from the specified production target. The proposed model objective is presented in Equation (1). The objective must satisfy several constraint functions to meet the suggested model’s goal. Constraint functions such as reserve constraints, slope constraints, mining constraints, processing constraints, and metal processing constraints are presented in Equations (2)–(6).

$$\max \frac{1}{S} \left\{ \sum_{s=1}^S \sum_{t=1}^T \sum_{i=1}^N c_{its} x_{it} - \sum_{s=1}^S \sum_{t=1}^T (v_t^{o-} d_{ts}^{o-} + v_t^{o+} d_{ts}^{o+} + v_t^{m-} d_{ts}^{m-} + v_t^{m+} d_{ts}^{m+}) \right\} \tag{1}$$

Subject to:

$$\sum_{t=1}^T x_{it} \leq 1 \quad i = 1, \dots, N \tag{2}$$

$$x_{it} - \sum_{\tau=1}^t x_{p\tau} \leq 0 \quad p \in P_i, \quad t = 1, \dots, T \tag{3}$$

$$\sum_{i=1}^N w_i x_{it} \leq \overline{W_{upper}} \quad t = 1, \dots, T \tag{4}$$

$$\sum_{i=1}^N w_i x_{it} \geq \underline{W_{lower}} \quad t = 1, \dots, T \tag{4}$$

$$\sum_{i=1}^N o_{is} w_i x_{it} - d_{ts}^{o+} \leq \overline{O_{upper}} \quad s = 1, \dots, S, \quad t = 1, \dots, T \tag{5}$$

$$\sum_{i=1}^N o_{is} w_i x_{it} + d_{ts}^{o-} \geq \underline{O_{lower}} \quad s = 1, \dots, S, \quad t = 1, \dots, T \tag{5}$$

$$\sum_{i=1}^N o_{is} m_{is} x_{it} - d_{ts}^{m+} \leq \overline{M_{upper}} \quad s = 1, \dots, S, \quad t = 1, \dots, T \tag{6}$$

$$\sum_{i=1}^N o_{is} m_{is} x_{it} + d_{ts}^{m-} \geq \underline{M_{lower}} \quad s = 1, \dots, S, \quad t = 1, \dots, T \tag{6}$$

where

c_{its} = Economic value of block i from simulation s for time t ;

x_i = Mining block of an open-pit mine, where $x_i \in X$ and X is the set of all blocks in a deposit;

d = Discounted rate;

N = The total number of blocks considered for scheduling;

i = Block index, $i = 1, \dots, N$;

T = The number of periods over which blocks are being scheduled;

t = period index, $t = 1, \dots, T$;

P_i = The set of predecessors of block i ; i.e., blocks that should be removed before i can be mined. Note that if block p is a predecessor of block i , then i is called a successor of p ;

s_i = The set of successors of block i ;

w_i = The weight of block i ;

$o_{is} = \begin{cases} 1 & \text{if block } i \text{ is ore block in simulation } s \\ 0 & \text{otherwise} \end{cases}$

m_i = The amount of metal in block i ;

S = The number of scenarios used to model geology uncertainties;

s = Scenario index, $s = 1, \dots, S$;

$\overline{W_{upper}}$ = The maximum amount of material at period t ;

$\underline{W_{lower}}$ = The minimum amount of material at period t ;

$\underline{O_{lower}}$ = Minimum ore required to feed the processing plant during period t ;

$\overline{O_{upper}}$ = Maximum ore processed in the plant during period t ;

$v_t^{o-} = \frac{v^{o-}}{(1+d_2)^t}$ = Unit shortage cost that can be associated with failure to meet \underline{O}_t during period t (v^{o-} is the undiscounted unit shortage cost, and d_2 represents the risk discount rate);

$v_t^{o+} = \frac{v^{o+}}{(1+d_2)^t}$ = Unit surplus cost incurred if the total weight of the ore blocks mined during period t exceeds \overline{O}_t ;

$\underline{M_{lower}}$ = Minimum amount of metal that should be produced during period t ;

$\overline{M_{upper}}$ = Maximum amount of metal that can be sold during period t ;

$v_t^{m-} = \frac{v^{m-}}{(1+d_2)^t}$ = Unit shortage cost associated with failure to meet \underline{M}_t during period t ;

$v_t^{m+} = \frac{v^{m+}}{(1+d_2)^t}$ = Unit surplus cost incurred if the metal production during period t exceeds \overline{M}_t ;

d_{ts}^{o-} = Shortage of ore at a discounted rate during period t in simulation s ;

d_{ts}^{o+} = Surplus of ore at a discounted rate during period t in simulation s ;

d_{ts}^{m-} = Shortage of metal for selling at a discounted rate during period t in simulation s ;

d_{ts}^{m+} = Surplus of metal for selling at a discounted rate during period t in simulation s .

3.5. Solving Stochastic Graph Closure Problem for Pit Optimization

It must be noted that the target function of Equation (1) with the constraints of Equation (2) only provides the ultimate pit. A parametric graph is suggested in this research to meet the limitations of Equations (3)–(6). The network flow algorithm will be used to solve a parametric open-pit graph problem with the maximum flow or minimum cut. The maximum cut algorithm’s purpose is to cut the arcs with the smallest capacity.

The proposed stochastic graph algorithm’s parametric formulation is as follows:

$$\Phi(\lambda) = \max \frac{1}{S} \sum_s \sum_{i=1}^N d_{i,s} x_i \tag{7}$$

where

$$d_{i,s} = \lambda * c_{i,s} \text{ if } c_{i,s} > 0; d_{i,s} = c_{i,s} \text{ otherwise}$$

$$\begin{aligned} x_i - x_p &\leq 0 \quad i = 1, \dots, N, p \in P_i \\ x_i &= 0 \text{ or } 1 \quad i = 1, \dots, N, t = 1 \end{aligned} \tag{8}$$

The algorithm begins with a small λ value and updates it at each iteration.

3.6. Repair Algorithm to Generate a Feasible Solution

It is understood that the different λ values can generate different-sized pits; but there is no guarantee that the generated pit will respect the set of constraints that have dropped (Equations (3)–(6)). Without respecting this set of constraints, it is not possible to generate a feasible schedule for the production period. To generate a feasible schedule, the following steps will be followed:

Step 1: The problem formulated in Equations (7) and (8) will be solved with a small λ value, and the value of λ will be updated at each iteration until all lower limit constraints (Equations (4)–(6)) are violated for all simulations, S . Assign the solution of this step as $\Phi(\lambda_k)$, where k is the number of iterations.

Step 2: The problem formulated in Equations (7) and (8) will be solved with $\lambda = \lambda_k$ value, and the value of λ will be updated at each iteration until all upper limit constraints (Equations (4)–(6)) are violated for all simulations, S . Assign the solution of this step as $\Phi(\lambda_m)$, where m is the number of iterations and $m > k$. A set of blocks, j , will be identified, such that $j \in \Phi(\lambda_m)$ and $j \notin \Phi(\lambda_k)$.

Step 3: The stochastic model will be formulated using these sets of j blocks incorporating the (Equations (4)–(6)) constraints. The updated stochastic model formulation can be presented as:

$$\max \sum_{j=1}^{N_1} \sum_{s=1}^S c_{j,s} x_j \tag{9}$$

Subject to:

$$x_j \in \{0, 1\}, j \in N_1 \tag{10}$$

$$x_j - x_p \leq 0 \quad j = 1, \dots, N_1, p \in P_j \tag{11}$$

$$\sum_{j=1}^{N_1} w_j x_j \leq \overline{W_{upper}} - W1 \tag{12}$$

$$\sum_{j=1}^{N_1} w_j x_j \geq \underline{W_{lower}} - W1$$

$$\sum_{j=1}^{N_1} o_{js} w_j x_j - d_s^{o+} \leq \overline{O_{upper}} - O1_s, s = 1, \dots, S \tag{13}$$

$$\sum_{j=1}^{N_1} o_{js} w_j x_j + d_s^{o-} \geq \underline{O_{lower}} - O1_s, s = 1, \dots, S$$

$$\sum_{j=1}^{N_1} o_{js} m_{js} x_j - d_s^{m+} \leq \overline{M_{upper}} - M1_s, s = 1, \dots, S \tag{14}$$

$$\sum_{j=1}^{N_1} o_{js} m_{js} x_j + d_s^{m-} \geq \underline{M_{lower}} - M1_s, s = 1, \dots, S$$

where

N_1 is the number of blocks belonging to solution $\Phi(\lambda_m)$ but does not belong to $\Phi(\lambda_k)$; $N_1 \leq N$, the computational time of SIP will be very less;

- W1 = Total weight of material in solution $\Phi(\lambda_k)$;
- O1_s = Total amount of ore at simulation *s* in solution $\Phi(\lambda_k)$;
- M1_s = Total amount of metal at simulation *s* in solution $\Phi(\lambda_k)$.

The minimum-cut technique will be applied to solve the following formulation, and the minimum-cut solution will be combined with $\Phi(\lambda_k)$ to generate a feasible solution. The proposed pseudo-code is used to achieve the goal of the proposed model.

Pseudo-code steps:

- Step 1: Formulation of stochastic mine production scheduling of mining blocks, $x_i, x_i \in X$.
- Step 2: Generate a graph problem (Φ_t) from a set time, *t*.
- Step 3: Solve the graph closure problem using a minimum-cut algorithm.
- Step 4: Choose parameter λ so that $\Phi(\lambda_k)$ violates all lower-bound constraints and $\Phi(\lambda_m)$ violates all upper-bound constraints; determine set *j* such that $j \in \Phi(\lambda_m)$ and $j \notin \Phi(\lambda_k)$.
- Step 5: Solve the stochastic problem for the set *j* by the minimum-cut algorithm. (Solution = $\Phi(\lambda_m)$).
- Step 6: Add $\Phi(\lambda_k)$ with $\Phi_\lambda = (\Phi(\lambda_m \cup \lambda_k))$ this is the solution of Φ_t .
- Step 7: Eliminate $x_i, x_i \in \Phi_t$ from set *X*; new set $X = \{x_j, x_j \notin \Phi_t\}; t = t + 1$.
- Step 8: If $t \leq T$, go to step 2 and repeat the process to find a new small pit.
- Step 9: If $t \geq T$ stop the process.

4. Results

The process for the uncertainty-based ultimate pit limit and pushback design is demonstrated in this section using a case study. The suggested method for pushback design and final pit limit has been applied to a copper deposit. The deposit is situated in a greenstone belt that dates back to the Archean period. The region is primarily composed of mafic lavas, with minor amounts of moderate to felsic volcanics. The geological dataset includes 185 drill holes with 10 m downhole composites in a 50 m × 50 m pseudo-regular grid, spanning a roughly rectangular region of 1600 × 900 m². A geostatistical analysis is used to define and model one mineralized domain using the geological information available. The simulation orebody model of the case study mine was generated using the direct block simulation technique [28]. The variogram analysis was used to determine the data’s spatial correlation. The deposit’s directional experimental variograms were determined in four different directions, with a spread of 22.5°, namely 0°, 45°, 90°, and 135°. The lag spacing for calculating the variograms along the strike for sulphides and oxides was 24 m and 32 m, respectively. Variograms along the downhole direction (−90° dip) were also calculated. All variograms were fitted with spherical variogram models. Except for the sulfide east along the downhole (dip −90°), all variograms were fitted by a single structure using a nugget model for copper. The deposit’s block model was estimated using the ordinary block kriging estimation technique. Due to the size of the selective mining unit (SMU), the projected block size is 20 m × 20 m × 10 m. For this investigation, simulated models were created. Within the mineralized zone, there are 9953 blocks with dimensions of 20 m × 20 m × 10 m. Equation (15) is used to calculate the block economic values of each block, which are then used to determine the ultimate pit limit and pushback design.

$$BEV = \begin{cases} Net\ revenue - MP - PP, & \text{if } Net\ revenue_i > PP \\ -MP_i & , \text{ otherwise} \end{cases} \tag{15}$$

where

$$Net\ revenue = T \times G \times REC \times (Price - Selling\ Price);$$

MP= mining price, PP = processing price, *T_i* = tonnage, *G_i* = grade, REC = recovery

The economic parameters from Table 1 were utilized to determine the block’s economic value.

Table 1. Economic parameters.

Parameters/Unit	Values
Copper price (US\$/lb)	2.0
Selling price (US\$/lb)	0.3
Mining price (\$/tonne)	1.0
Processing price (\$/tonne)	9.0
Recovery (%)	0.9

4.1. Ultimate Pit Generation

The directed graphs are created utilizing the block economic value of simulated orebody models to construct the ultimate pit. In this article, the block economic value is estimated as an undiscounted value. The simulation takes place in the mineralized zone; some waste blocks are added to the non-mineralized zone to establish a smooth topography and create a standard 3D orebody model. A directed graph is constructed, as mentioned in Section 3.2, with ore blocks linked to the source node and waste blocks linked to the sink node. To preserve slope restrictions, an infinite capacity arc is made for underlying blocks to overlying blocks. Since the study mine has a 45° slope angle, infinite capacity arcs are carried from an underlying block to nine overlying neighbor blocks. The infinite arc capacities are maintained by selecting a high positive number. The push re-label maximum flow algorithm is used to create the ultimate pit after the graph has been created. Two sections of the final pit created using the suggested method are shown in Figure 6.

Pushback Design and Mine Production Scheduling

The parameterization of the minimum-cut algorithm was carried out to produce a sequence of nested pits. The economic worth of the ore blocks was scaled using the parameter λ , as explained in Section 3.3, to produce eight pushbacks. To reduce the distance between pushbacks, the number of pushbacks in this study was determined through trial and error. Eight pushbacks were found to decrease the distance between them. The pushback sequences for the mine under study are shown in two sections in Figure 6.

It is necessary to evaluate the per-year mine production schedule to determine the discounted cash flows and total net present value for the case study. Production scheduling was carried out using the technical parameters listed in Table 1. Twenty simulated orebody models were used to create a model of the deposit, and from this model, the total ore quantity and ultimate pit limit were estimated to set the yearly production plan. The mine's life is estimated to be 8 years. As a result, the production goal was set by planning the model for 8 years. The considered cut-off grade for copper is 0.3%. To fulfill the production goals, bench-wise scheduling was used (i.e., extract the first bench of the first pushback, then the second bench of the first pushback, and so on, until the first pushback is extracted, and then repeat for the following pushback). Figure 7 displays the sections for an 8-year scheduling scenario. Figures 8–12 show the calculated (minimum, maximum, and average) mining capacity, ore production capacity, cumulative metal quantities, and comparison of the proposed model cumulative undiscounted cash flow with the deterministic model.

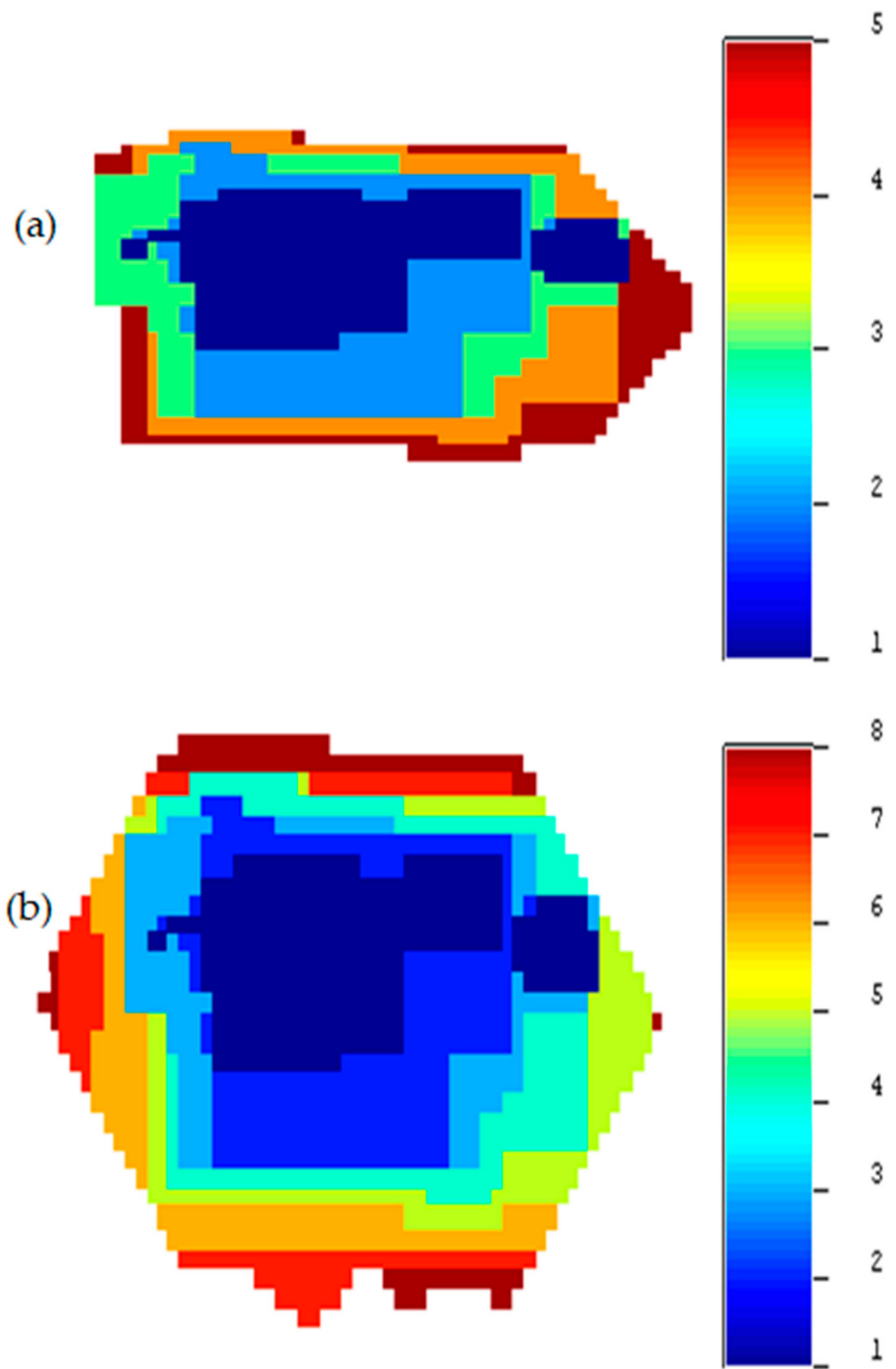


Figure 6. (a) Section view of the ultimate pit for 5 periods. (b) Top view of ultimate pit for 8 periods.

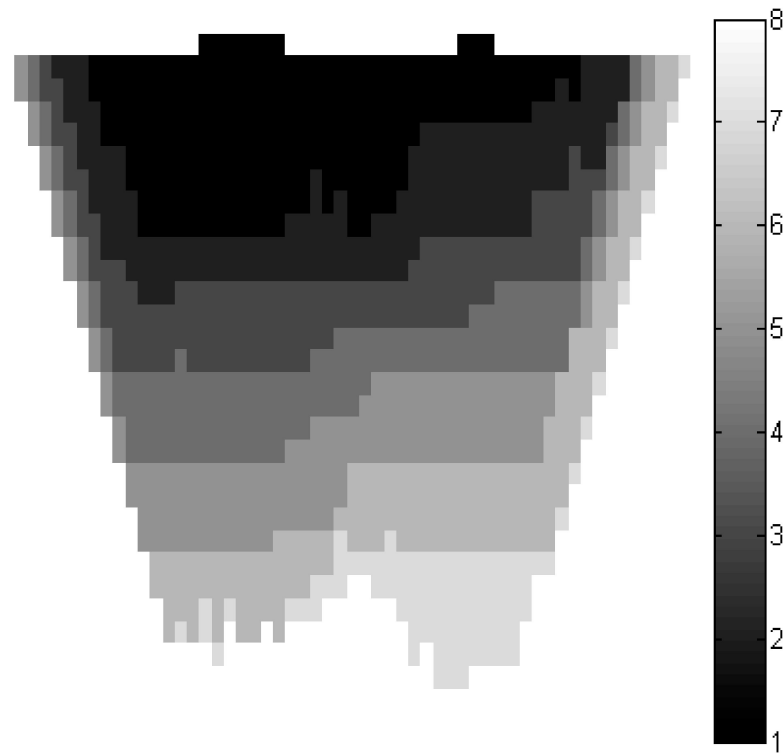


Figure 7. Production schedule of the deposit mine.

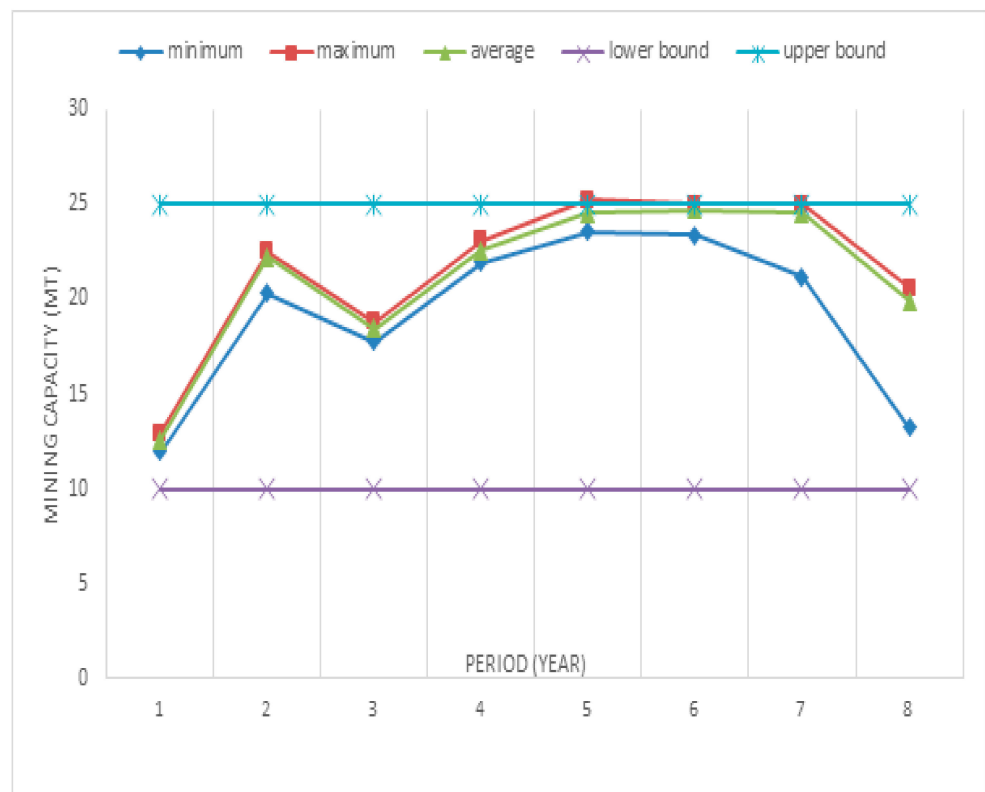


Figure 8. Mining capacity for production schedule.

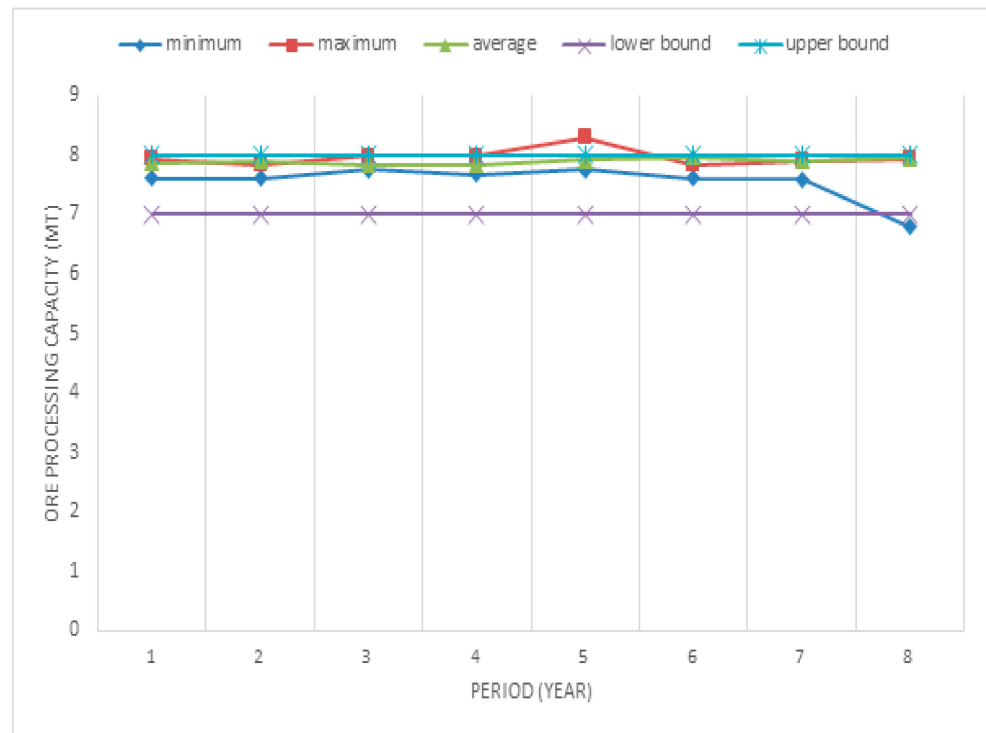


Figure 9. Ore production capacity of the production schedule.

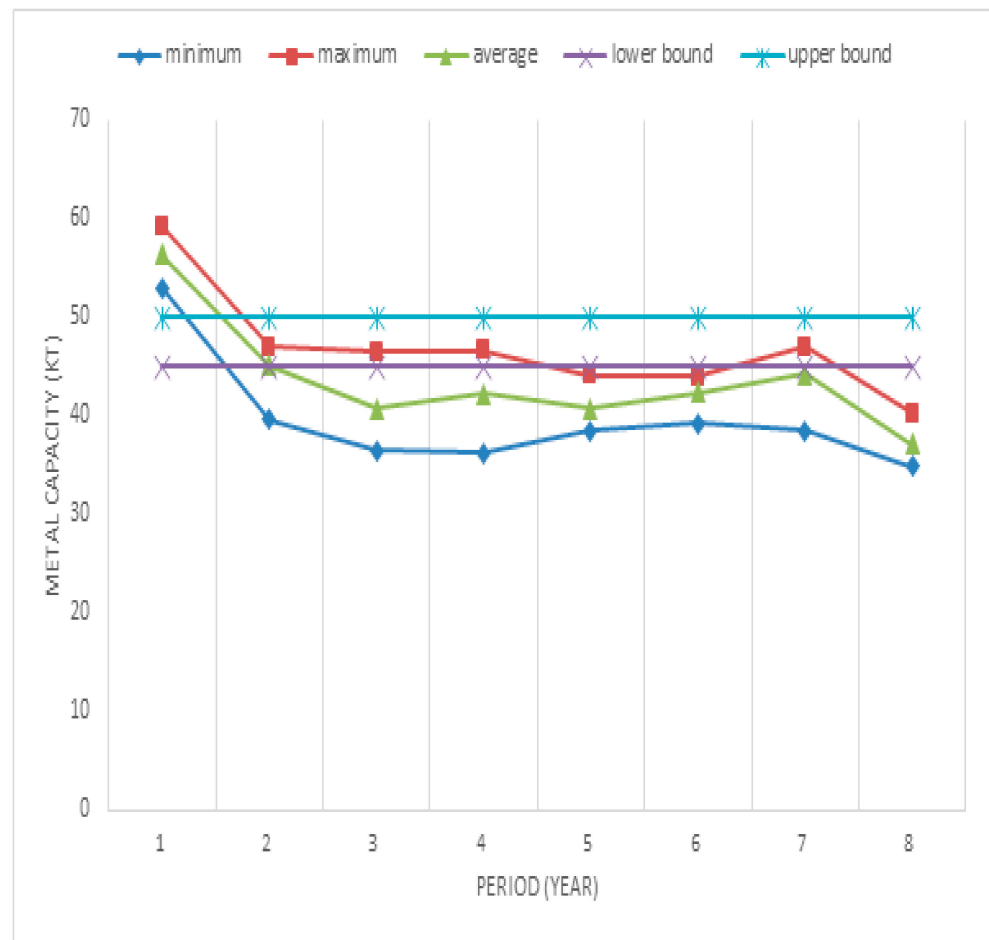


Figure 10. Metal production capacity of case study deposit.

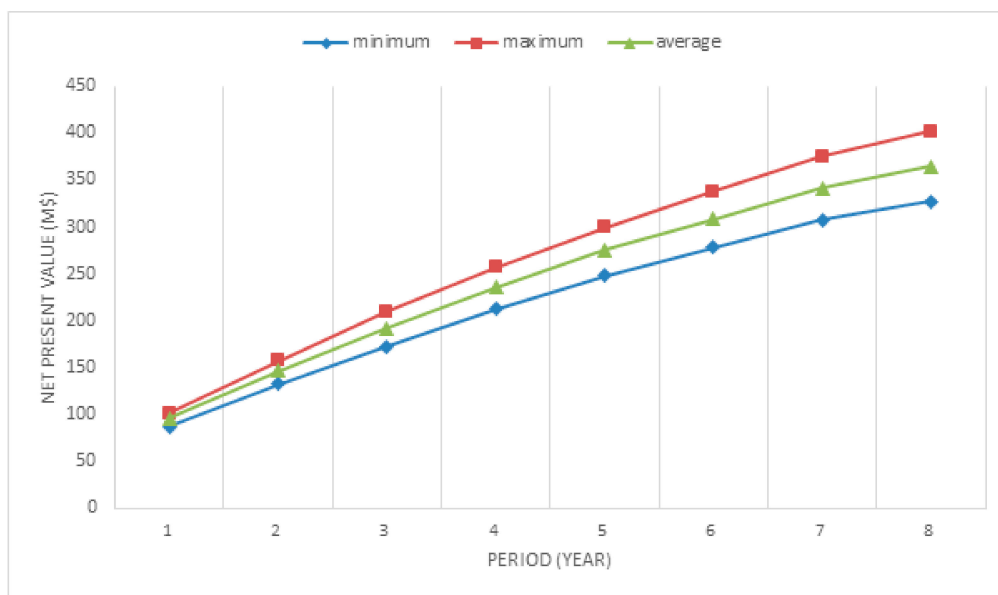


Figure 11. Undiscounted cumulative cash flow of case study deposit.

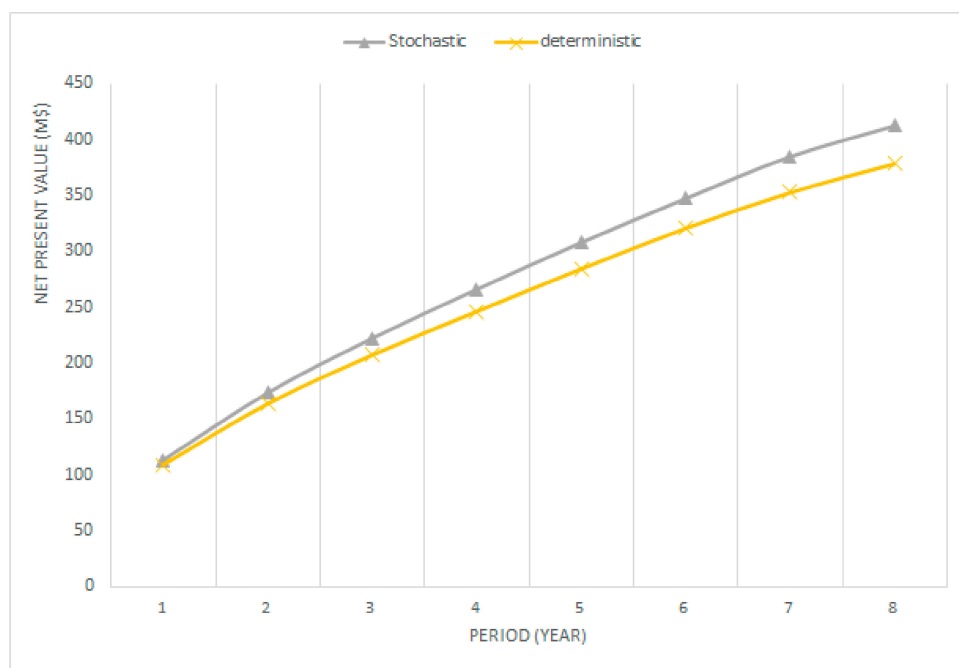


Figure 12. Comparison of undiscounted cumulative cash flow with a deterministic model.

5. Conclusions and Future Scope

To jointly integrate various orebody models, the minimum-cut graph approach was employed in this research together with the uncertainty-based ultimate pit limit and pushback design. The recommended method shows that it is quite simple to develop a minimum-cut network flow model that incorporates uncertainty and can handle uncertainty in the economic value of the mining blocks being scheduled. The example given here is based on block values that are uncertain because of the metal content, but the same method can be easily applied to account for demand uncertainties (commodity cost, exchange rate), as well as uncertainties in mining price, processing price, metal recovery, or any other input used to determine the block economic value. The key benefit of the suggested technique is that it is computationally very quick, making it possible to consistently incorporate many

uncertainties in the optimization process. On a copper deposit with low variability, the suggested approach was successfully tested. The most significant conclusion was that, similar to other stochastic mine planning methods, the difference in net present value generated was 10% higher than with the equivalent deterministic (traditional) approach. We will run more field testing to examine the network flow approach's potential in uncertain situations.

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